Electrostatics

The Physics of stationary electric charges, i.e. charges at rest called electrostatics

- The charges are acquired by the bodies on rubbing with each other
- Like charges repel and unlike charges attract each other
- Electrons have negative charge. e = - 1.6 X 10^{-19} Coulomb
- Protons have positive charge. e = 1.6 X 10^{-19} Coulomb

Quantization of charge

All physical existing charge in the universe is in integral multiples of electronic charge.

Charge Q = ne

charge exists in discrete packets rather than in continuous amounts. Hence the charge is quantized.

Conservation of charge

The total electric charge in an isolated system never changes. or

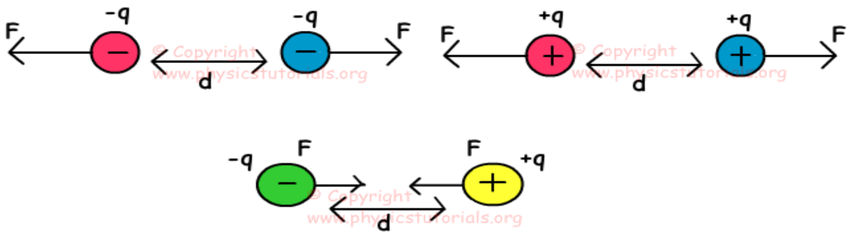
The algebraic sum of the electric charges remains constant in a closed system implies that charge can neither be created nor destroyed

Electric field

The region surrounding an electric charge or a group of charges in which another charge experiences a force is called the Electric field

Coulomb's inverse square law

The electrostatic force of attraction or repulsion between two charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them .



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 \times q_2}{r^2}$$ Here ε_0 *is called permitivity of free space or air*

When the charges are placed in a medium of dielectric constant k, then

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

Intensity of electric field (E)

The intensity of electric field at a point in the field is defined as the force experienced by a unit positive charge placed at that point.

Let 'F' be the force experienced by a test charge q_0 placed at a point in the electric field.

The intensity of electric field E = $\frac{F}{q_0}$ newton/coulomb

E is a vector quantity. Its direction will be the same as the direction of force.

Intensity of electric field due to a point charge

Consider an isolated point charge + q coulomb is placed 'O' in air. A test charge q_0 place at 'P'.

According to Coulomb's law, the electric force acting on q_0 is given

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q \times q_0}{r^2} N \text{ --------- } (1)$$

We know that the intensity of electric field at any point is given by

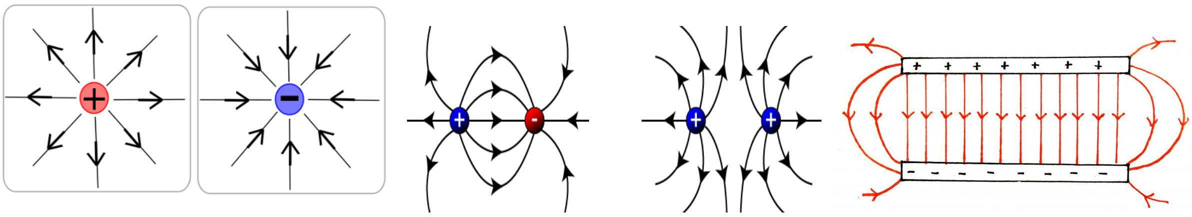
$$E = \frac{F}{q_0} \text{ ----------- } (2)$$

From equation (1) and (2)

E = $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} N/C$

Electric field lines

An electric field line is that imaginary smooth drawn in an electric field along which a free isolated positive charge will move. The tangent at any point on this curve gives the direction of the electric field at that point. The intensity of electric field at any point can also be defined as the number of lines of force passing through unit area around that point normally



Properties of Electric field lines:

1. The electric lines of force start from a positive charge and end on a negative charge.
2. The tangent drawn at any point on the line of force gives the direction of the electric field at that point.
3. No two lines of force intersect each other. The reason is that if they do so, then at the point of intersection two tangents can be drawn, i.e., two directions of force are possible at that point which is impossible.
4. The number of lines of force crossing the unit area in a normal direction is proportional to the electric intensity.
5. The lines of force have a tendency to contract in length. This explains the attraction between opposite charges.
6. The lines of force have a tendency to separate from each other in direction perpendicular to their lengths. This explains repulsion between like charges.
7. The lines of force are perpendicular to conducting surfaces at points close to them.
8. The electric lines of force are perpendicular to equipotential surfaces.

Electric Flux (Φ)

The electric flux through a surface placed inside an electric field represents the total number of electric lines of force crossing the surface in a direction normal to the surface.

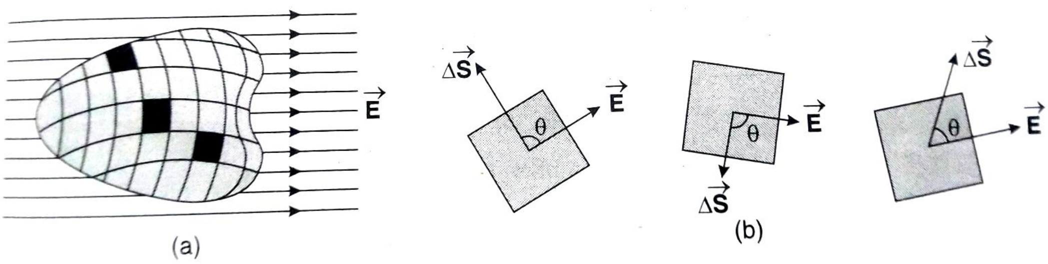


Fig. (3) (a) Surface immersed in electric field
(b) Enlarged view of the three squares on surface.

Let us consider an electric field. An arbitrary closed surface immersed in this field. Let the surface be divided into a number of elementary squares. Each square on the surface may be represented by a vector ΔS, whose magnitude is equal to its area and the direction taken as the outward normal drawn on the surface.

Let E be the electric field vector acting on the surface.

The scalar product $E.\Delta S$ is defined as the electric flux for the surface.

The total flux $\Phi_E = \oint E.\Delta S \;=\; E.S$

If θ is the angle between E and ΔS

$E.\Delta S = EdS\,cos\theta$

$\Phi_E = \oint E.dS \;=\; \oint EdS\,cos\theta \;=\; E\,cos\theta \oint dS \;=\; EA\,cos\theta$

Note: For a closed surface Φ_E is taken positive if the lines of force point outward everywhere and negative if they point inward.

Gauss's Law

Gauss's law states that total normal electric flux Φ_E over a closed surface is $(\frac{1}{\varepsilon_0})$times the total charge 'Q' enclosed within the surface.

$\Phi_E = \oint E.dS \;=\; \oint EdS\,cos\theta \;=\; (\frac{1}{\varepsilon_0})Q$

Here ε_0 is the permittivity of the force space.

Proof:

i) when the charges within the surface

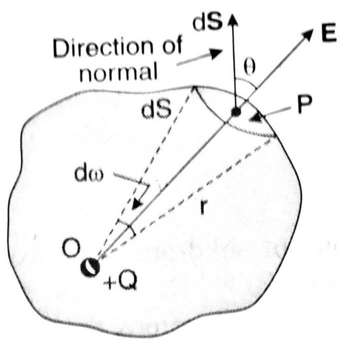
Let a charge '+Q' be placed at 'O' within a closed surface. Now take a small area dS around P. The normal to the surface dS is represented by a vector dS, which makes an angle θ with the direction of electric field E along OP.

The electric flux $d\Phi_E$ outwards through the area dS is given by

$d\Phi_E = E.dS \;=\; EdS\,cos\theta$ ----------- (1)

whereθ is an angle between E and dS

For Coulomb's law, the electric intensity E at a point P distance r from a point charge Q is given by $E \;=\; \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$--------- (2)



From equations (1) and (2), we get

$d\Phi_E = \frac{Q}{4\pi\varepsilon_0}(\frac{dS\,cos\theta}{r^2}) \;=\; \frac{Q}{4\pi\varepsilon_0}d\omega,$

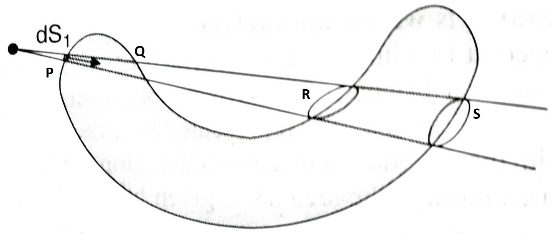
$$here\ solid\ angle\ d\omega \;=\; \frac{dS\,cos\theta}{r^2}$$

The total flux $\Phi_E = \oint \frac{Q}{4\pi\varepsilon_0}d\omega \;=\; \frac{Q}{4\pi\varepsilon_0}\oint d\omega$ Where $\oint d\omega$is the solid angle subtended by the whole surface at O. This is equal to 4π.

$\therefore\ \Phi_E = \frac{Q}{4\pi\varepsilon_0} \times\ 4\pi \;=\; \frac{Q}{\varepsilon_0}$

ii) When the charge is outside the surface

Let a point charge +Q be situated at point O, outside the closed surface. Now, a cone of solid angle d**ω** from O cuts the surface areas dS_1, dS_2,dS_3 and dS_4 at points P,Q,R and S respectively.



The electric flux at P through area dS_1 = $(\frac{-Q}{4\pi\varepsilon_0})d\omega$

The electric flux at Q through area dS_2 = $(\frac{+Q}{4\pi\varepsilon_0})d\omega$

The electric flux at R through area dS_3 = $(\frac{-Q}{4\pi\varepsilon_0})d\omega$

The electric flux at S through area dS_4 = $(\frac{+Q}{4\pi\varepsilon_0})d\omega$

The electric flux = $(\frac{-Q}{4\pi\varepsilon_0})d\omega + (\frac{+Q}{4\pi\varepsilon_0})d\omega + (\frac{-Q}{4\pi\varepsilon_0})d\omega + (\frac{+Q}{4\pi\varepsilon_0})d\omega = 0$

The total electric flux over the whole surface due to an external charge is zero

Differential form of Gauss's law

According to Gauss's law

$\oint E.dS = (\frac{Q}{\varepsilon_0})$ or $\varepsilon_0 \oint E.dS = Q$ ---------- (1)

Let a charge Q be distributed uniformly over a volume V and ρ be the density of charge,

Then $Q = \int\int\int \rho dV$ ------ (2)

From equations (1) and (2) $\varepsilon_0 \oint E.dS = \int\int\int \rho dV$ ---------- (3)

According to divergence theorem $\oint E.dS = \int\int\int div\, E dV$ ------- (4)

Substituting equation (4) in (3) $\varepsilon_0 \int\int\int div\, E dV = \int\int\int \rho dV$------------(5)

Equation (5) is true for any arbitrary volume. Hence integrands must be equal.

$\varepsilon_0\ div\, E = \rho \Rightarrow div\, E = \frac{\rho}{\varepsilon_0}$

$\nabla.E = \frac{\rho}{\varepsilon_0}$ This is the differential form of Gauss's law

In vacuum $D = \varepsilon_0 E \Rightarrow E = \frac{D}{\varepsilon_0}$

div D = ρ or ∇.D = ρ

Applications of Gauss's law

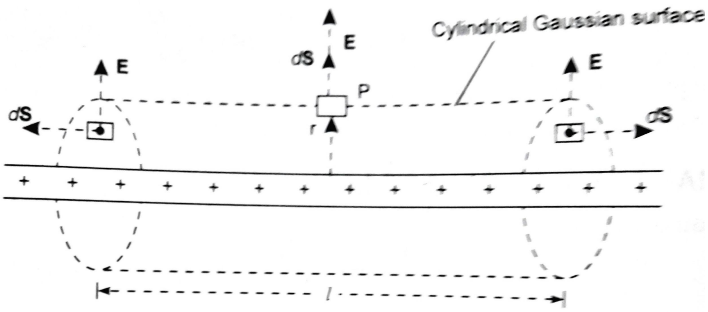
1. **Electric field due to an infinitely long charge distribution**

Consider an infinitely long line charge having a linear density λ.

To calculate electric field at a point 'P' at a distance 'r' from the line charge.

As the charge distribution is linear, we construct a cylindrical Gaussian surface of radius 'r' and length 'l'.

All the points on the curved surface are at the same perpendicular distance from the line charge.



The direction of the field at any point on the curved surface is normal to the cylindrical surface.

At the ends of the cylinder E will be perpendicular to dS.

The net flux through the cylindrical Gaussian surface is

$\Phi = \oint E.dS = \oint_{left\ ent} E.dS + \oint_{right\ end} E.dS + \oint_{curved\ surface} E.dS$

At both the ends of the Gaussian cylinder, E is perpendicular to the axial area vector dS.

Hence E.dS = 0

On the curved surface, E is in the direction of dS. So, E.dS = E dS cosθ = E.dS

$\Phi = 0 + 0 + \oint_{curved\ surface} E.dS = E.\ 2\pi rl$ $here \oint dS = 2\pi rl = surface\ area\ of\ the\ cylinder.$

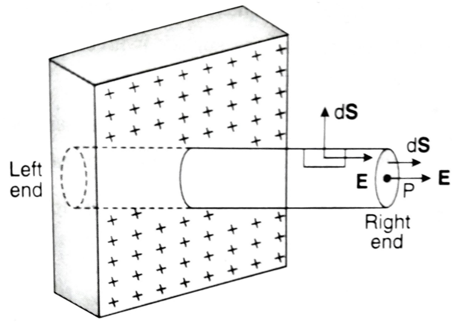
The charge enclosed by Gaussian surface is q = λl

According to the Gauss's law

$$\Phi = \frac{q}{\varepsilon_0} \quad \Rightarrow \quad E.2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

This gives the electric field due to an infinitely long charge distribution..



Electric field due to infinite conducting sheet of charge

A charged conducting surface of charge density σ. To determine the electric field at a point near the surface and outside the conductor, construct a cylindrical Gaussian surface. The direction of the electric field near the surface is perpendicular to the surface as the conducting surface is an equipotential surface.

Electric flux through the cylindrical Gaussian surface results from the two ends and curved surface of the cylinder. At the right, E is parallel to dS and at the left end there is no electric field. Therefore, the flux through the two ends are E.dS and zero respectively. The electric flux through the curved surface is zero as E and dS are perpendicular.

$$\Phi = \int_{right\ end} E.dS + \int_{left\ end} E.dS + \int_{curved\ surface} E.dS$$

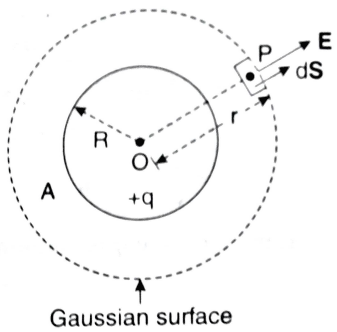
$$\Phi = ES + 0 + 0 = ES \qquad \Rightarrow ES = \frac{q}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{\sigma}{\varepsilon_0}$$

Electric field due to an uniformly charged sphere

Case (i): At a point outside the charged sphere

Consider a sphere of radius R with centre O. Let a charge q be uniformly distributed over it. Suppose P be an external point at a distance r from the centre of the sphere. We shall find the electric field at this point. For this purpose we construct a Gaussian surface of radius OP which is concentric with sphere A.

The electric flux through the entire Gaussian surface is given by



$$\Phi_E = \int E.dS = E\int.dS = E(4\pi r^2)$$

According to Gauss's law $\quad E(4\pi r^2) = \left(\frac{q}{\varepsilon_0}\right)$

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}\ newton/coulomb$$

Case (ii): At a point on the surface

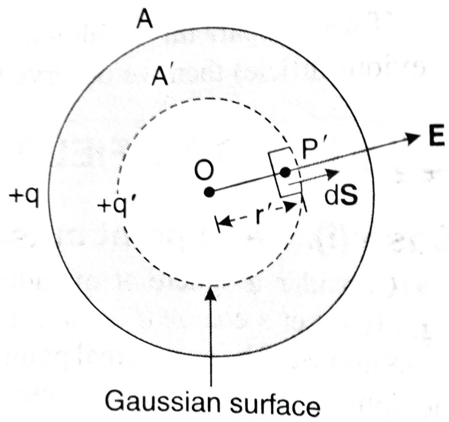
when the point P lies on the surface of the sphere, then r = R.

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{R^2}$$

Case (iii). At a point inside the charged sphere.

To find the electric field at a point P, which is inside the charged sphere at a distance r from the centre.

The electric flux through the entire surface is given by



$$\Phi_E = \int E.dS = E\int.dS = E(4\pi r^2)$$

The total charge enclosed by the Gaussian surface

$$q = \frac{4}{3}\pi R^3 \times \rho$$

$$\rho = \frac{q}{\frac{4}{3}\pi R^3} \Rightarrow \frac{3q}{4\pi R^3}$$

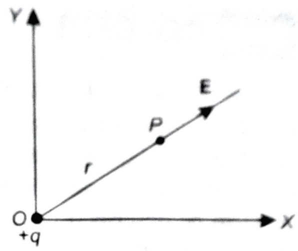
Charge enclosed in Gaussian surface $\frac{4}{3}\pi r^3 \times \frac{3q}{4\ \pi R^3} = q(\frac{r}{R})^3$

From Gauss's law $E(4\pi r^2) = \frac{1}{\varepsilon_0} \times q\,(\frac{r}{R})^3 \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \times \frac{qr}{R^3}$

Conservative nature of electric field

We know that the electric field E at a distance r due to a point charge +q placed at origin O

$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$



Further , consider any two arbitrary points A and B in the electric field region of charge +q.

The potential difference between two points A and B is given by

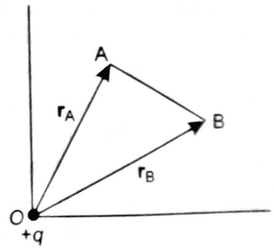
$\int_{r_A}^{r_B} E.. dr = \frac{1}{4\pi\varepsilon_0}\int_{r_A}^{r_B} \frac{q}{r^2} dr \Rightarrow \int_{A}^{B} E.. dr = -\frac{1}{4\pi\varepsilon_0}[\frac{q}{r_a} - \frac{q}{r_b}]$

Where r_A and r_B are position vectors of points A and B respectively.

If A and B are same points, the $r_A = r_B$

Then $\oint_c E.. dr = 0$

Applying Stoke's theorem, we get



$\oint_c E.. dr = \oint_s (\nabla \times E) . ds = 0 \Rightarrow \nabla \times E = 0$

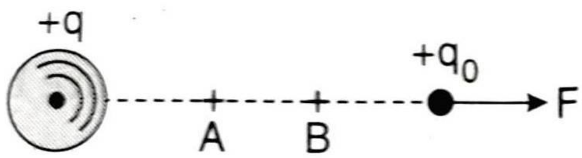
Irrotational field

The general meaning of curl is rotation. When the curl of a vector field is zero, it means that no rotation is attached with E. On the other hand, if curl E is non zero, it means that rotation is attached with vector E. Therefore, curl E = 0, then E is an irrotational field.

Potential difference

The work done in moving unit positive charge from one point to another point in an electric field is called potential difference between the two points.

$$V_A - V_B = \frac{W}{q_0}$$



The ratio of work done in taking a test charge from one point to another point in an electric field to the magnitude of the test charge is defined as the electric potential difference between these points.

Electric potential

The work done in moving a unit positive charge from infinity to a point against the electric force of the field is called the electric potential at that point

$$V_A = \frac{W}{q_0}$$

Electric potential at a point in the electric field is defined as the work done by an external agent in carrying a unit positive test charge from infinity to that point against the electric force of the field

Equipotential surfaces

Equipotential surfaces are those surfaces that have the same potential at all points

Properties:

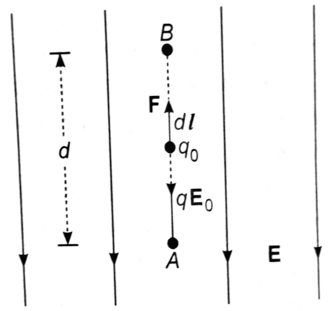
1. Electric field along the equipotential surface is zero.
2. The electric field is always normal to the surface
3. The work done in moving a charge on the equipotential surface is zero.
4. When the charge is infinite, the equipotential surface is plane.
5. The equipotential surfaces act as wave- fronts in optics.

Potential as a function of electric field

Suppose a positive test charge q_0 is moved by an external agent without acceleration between two points A and B in the electric field. The electric force on the charge q_0 is q_0 E. The direction of this force is in the direction of electric field strength E, which points downwards. To move the charge in the upward direction by an external agent, an equal and opposite force $-q_0$ E must be applied. Let the charge q_0 be moved through a small distance dl by the external agent.

The work done dw by the external agent is given by dw = F.dl = $-q_0$ E.dl

Therefore, the total work done W_{AB} in carrying q_0 from A to B will be



$$W_{AB} = \int_A^B - q_0 E.dl = - q_0 \int_A^B E.dl$$

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

Potential difference between two points A and B will be

$$V_B - V_A = - \int_A^B E.dl$$

If the reference point A is taken at infinity, the $V_A = 0$

$$V_B - V_A = - \int_\infty^B E.dl$$

The electric potential at a point in the electric field can be expressed as a line integral of the electric field.

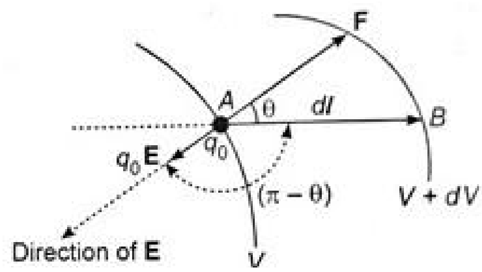
Relation between electric potential (V) and electric field (E) or

Field as the gradient of potential

Consider two surfaces of potential V and V + dV in an electric field. Let positive test charge q_0 at a point A on the surface V be moved to the point B on the surface V +dV along the vector displacement dl by any external agent. The force experienced by the test charge q_0 at A due to the electric field will be q_0 E. This force is in the direction of E, which is at right angles to the surface V. In order to move the test charge q_0 without acceleration, the external agent must apply an equal and opposite force F.

F = $-q_0$ E

The work done by the external agent to move the test charge from A to B along dl is given by



dw = F.dl $\Rightarrow dw = - q_0 E.dl \quad \Rightarrow \frac{dw}{q_0} = E.dl$

We know that $\frac{dw}{q_0}$ = dV

dV = $- E.dl$ --------- (1)

Let the coordinates of A and B be (x,y,z) and (x+dx, y+dy, z+dz) respectively. The potential difference dV between A and B can be expressed as

$$dv = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$= [i\frac{\partial V}{\partial x} + j\frac{\partial V}{\partial y} + k\frac{\partial V}{\partial z}] . (dx i + dy j + dz k)$$

(grad V) . dl = ∇V .dl -------- (2)

$[i\frac{\partial V}{\partial x} + j\frac{\partial V}{\partial y} + k\frac{\partial V}{\partial z}]$ = grad V and d$x i$ +d$y j$ + dz k is the displacement vector dl between A and B.

Comparing equations (1) and (2), we get

qE = - grad V = -∇V -----(3)

If E_x, E_y *and* E_z be the components of E, then equation (3) can be written as

$$E_x i + E_y j + E_z k = -[\frac{\partial V}{\partial x}i + \frac{\partial V}{\partial y}j + \frac{\partial V}{\partial z}k]$$

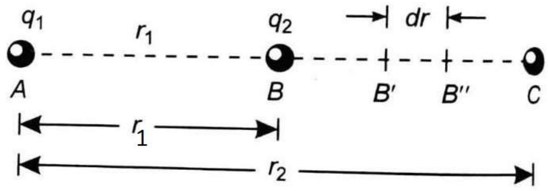
$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z - \frac{\partial V}{\partial z}$$

The electric intensity at a point in the electric field is equal to the negative potential gradient in that direction.

Potential energy of system of charges

Consider a system of two charges $q_1$and q_2 separated by distance r_1. The charge q_1 is fixed at point$A$. Let the charge q_2 be taken from position B to position C along the line ABC.

Let $AC = r_2$. Consider a small displacement of charge q_2 from a distance r to $(r + dr)$ between B and C (i.e., from position B' to B'').



The electric force on charge q_2 at position B' is

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\ towards\ AB \qquad .... (1)$$

The work done by this force for a small displacement dr is

$$dW = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\ dr \qquad .... (2)$$

The total work done when charge $q_2$is moved from B to C is

$$W = \int_{r_1}^{r_2} \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\ dr = \frac{q_1 q_2}{4\pi\varepsilon_0}[\frac{1}{r_1} - \frac{1}{r_2}] \qquad .... (3)$$

The change in potential energy is

$$U(r_2) - U(r_1) = -W = \frac{q_1 q_2}{4\pi\varepsilon_0}[\frac{1}{r_2} - \frac{1}{r_1}] \qquad .... (4)$$

The potential energy of two charge systems is taken as zero when they have infinite separation. Thus

$$U(r) = U(r_2) - U(\infty)$$

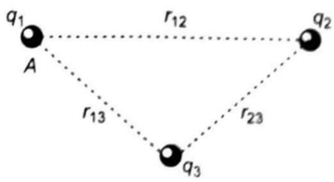
$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}[\frac{1}{r_2} - \frac{1}{\infty}]$$

$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}. \qquad .... (5)$$

Or

Eq(5) give the potential energy of a system composed of two charges q_1 *and* q_2.

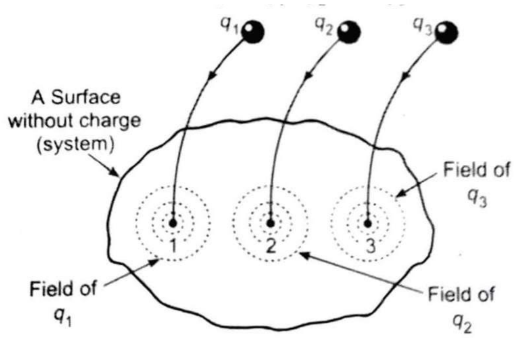
For a system of three charges the potential energy is given by



$$U = \frac{1}{4\pi\varepsilon_0}[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_3}{r_{13}}] \qquad .... (6)$$

ENERGY DENSITY IN ELECTROSTATIC FIELD

Let a test charge be moved from a point of lower potential to a point of higher potential in an electric field. In this process, the external agent that moves the point charge has to do some work on it. As a result, the potential energy of the system is increased, i.e., the system has a positive energy. On the other hand, if the test charge is moved from higher potential to lower potential in an electric field, then the electric field does work on the test charge. Therefore, the potential energy of the system is decreased, ie, there is a loss of potential energy. This energy is supplied to the test charge.



Now, we shall calculate the potential energy of a system consisting of a number (say n) of charges. First of all we shall find the potential energy of the system due to three charges, q_1, q_2 and q_3 and then generalize it for n charges. For this purpose, we have to calculate the work done by external sources in positioning these charges on a surface as shown in fig. The surface is assumed to be without charge.

In bringing the charge q_1 from infinity to position1, no work is done because the field does not exit on the system.so, $W_1 = 0$

Now consider that charge q_2 is taken from infinity to position 2. The movement of this charge will take place in the field of charge q_1. This requires a certain amount of work. The work done is given by $W_2 = charge \times potential = q_2 V_{21}$ where V_{21} is the potential at the location of q_2 due to q_1.

Similarly, in bringing the charge q_3 from infinity to position 3, the movement will b in the field of charges q_1 and q_2. The work done is given by $W_3 = q_3 V_{31} + q_3 V_{32}$, where V_{31} and V_{32} are the potentials at position 3 due to the charges q_1 and q_2 respectively.

$$total\ work\ done = Potential\ energy\ of\ the\ field$$

$$W_E = 0 + (q_2 V_{21}) + (q_3 V_{31} + q_3 V_{32}) \quad \text{.....................} (1)$$

Consider that the three charges are brought in the reverse order, $i, e.,$ charge q_3 is taken first and then charge q_2 and finally charge q_1. The total work done in this situation is given by

$$W_E = 0 + (q_2 V_{23}) + (q_1 V_{12} + q_1 V_{13}) \quad \text{.....................} (2)$$

Adding eq(1) and eq(2), we get

$$2W_E = (q_1 V_{12} + q_1 V_{13}) + (q_2 V_{21} + q_2 V_{23}) + (q_3 V_{31} + q_3 V_{32})$$

or $$2W_E = q_1(V_{12} + V_{13}) + q_2(V_{21} + V_{23}) + q_3(V_{31} + V_{32})$$

or $$2W_E = q_1 V_1 + q_2 V_2 + q_3 V_3 \quad \text{................} (3)$$

Where $V_1 = V_{12} + V_{13} = total\ potential\ of\ charge\ q,$

$$\therefore \quad W_E = \frac{1}{2}[q_1 V_1 + q_2 V_2 + q_3 V_3] \quad \text{...........................} (4)$$

If we consider a number of charges, then in general

$$W_E = \frac{1}{2}\sum_{i=1}^{i=n} q_1 V_1 \quad \text{..........................} (5)$$

General Expression

To get the **general expression** for energy stored in a region of continuous charge distribution, we replace the summation by integration and we substitute q by $\rho\ dV$

$$\therefore \quad W_E = \frac{1}{2}\int\int\int_V \rho\ dV \quad \text{....................} (6)$$

According to Gauss's law, $\vec{\nabla}.D = \rho$

So, $$W_E = \frac{1}{2}\int\int\int_V (\vec{\nabla}.D)V\, dV \quad .... (7)$$

Where V is scalar and D is a vector field.

To obtain the value of volume integral, we use the following vector identity

$$\vec{\nabla}.(V\,D) \equiv V(\vec{\nabla}.D) + D.(\vec{\nabla}V) \quad .... (8)$$

or $$V(\vec{\nabla}.D) \equiv \vec{\nabla}.(V\,D) - D(\vec{\nabla}V)$$

Substituting this value in eq(7). We get

$$W_E = \frac{1}{2}[\int\int\int_V (\vec{\nabla}.VD)\, dV - \int\int\int_V D.(\vec{\nabla}V)\, dV].$$

Using divergence theorem, the first volume integral of this equation can be changed into closed surface integral, Now, we have

$$W_E = \frac{1}{2}[\oint_s V\,D\,dS - \int\int\int_V D.(\vec{\nabla}V)\, dV \quad .... (9)$$

The surface integral over this closed surface is zero.

$$W_E = -\frac{1}{2}\int\int\int_V D.(\vec{\nabla}V)dV = \frac{1}{2}\int\int\int_V D.(-\vec{\nabla}V)dV$$

$$= \frac{1}{2}\int\int\int_v (D.E)\, dV \qquad (\because E = -\vec{\nabla}V)$$

$$= \frac{1}{2}\int\int\int_v (\varepsilon_0.E.E)\, dV \qquad (\because D = \varepsilon_0 E)$$

or $$W_E = \frac{1}{2}\int\int\int_v \varepsilon_0 E^2 dV \quad .... (10)$$

This is known as **energy density** or **energy stored** for any field **E**.

POTENTIAL DUE TO A POINT CHARGE

Consider a point charge $+q$. Its electric field **E** is outward along a radial line. The aim of this article is to calculate the potential at a point B situated at a distance r_b from the charge + q. For this purpose we select two points A and B along the radial line (for convenience). Let a test charge q_0 be moved from reference point A to B.

The force exerted by the field of charge q on test charge q_0 is q_0E.

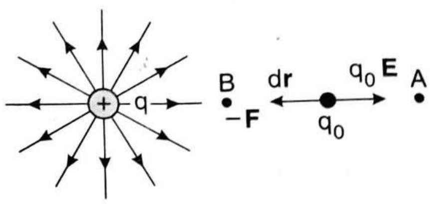
Now to move the test charge q_0 towards B, a force $-q_0E$ must be applied. The work done by external agent to move the charge q_0 through a small distance dr is given by

$$dW = q_0E.dr = q_0E\,dr\,cos\,180^o = -q_0E\,dr$$

Further $$E = \frac{1}{4\pi\varepsilon_0 r}.\frac{q}{r^2} \qquad \because dW = -\frac{1}{4\pi\varepsilon_0}.\frac{q\,q_0}{r^2}.dr$$

Now the total work done in moving the test charge from A to B

$$W_{AB} = \int_{r_A}^{r_B} -\frac{1}{4\pi\varepsilon_0}.\frac{q\,q_0}{r^2}.dr$$



When r_A is the distance of point A from q.

$$= -\frac{q\,q_0}{4\pi\varepsilon_0}[-\frac{1}{r}]_{r_A}^{r_B} = \frac{q\,q_0}{4\pi\varepsilon_0}[\frac{1}{r_B} - \frac{1}{r_A}].$$

So the potential difference between two points will be

$$V_B - V_A = \frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right].$$

To find the potential at point B, the reference point A is taken at infinity so that $V_A = 0$. Hence,

$$V_B = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r_B}$$

On dropping the suffix, the required expression becomes

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

POTENTIAL (V) FOR SPHERICAL CHARGE DISTRIBUTION FROM ELECTRIC FIELD (E)

(a) When point lies outside the sphere

Consider a conducting charged spherical conductor with centre O and the radius R.

Let q be the total charge on the sphere. The charge is distributed on the spherical surface.

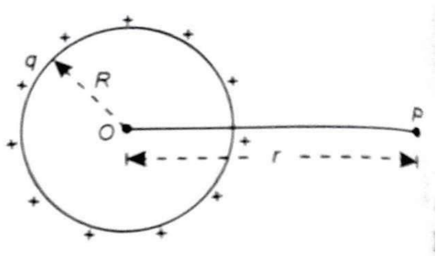
The aim of this article is to find the potential at a point P distant r from centre O of the sphere $i.e.,\ r > R$

The electric field intensity at point P is

$$E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2} \quad \text{.......... (1)}$$

We know that

$$V = -\int_A^B E.dr \quad \text{.............................. (2)}$$



$$\therefore \quad V = -\int_0^r \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}\,dr = -\frac{q}{4\pi\varepsilon_0}\int_0^r \frac{1}{r^2}dr$$

$$or \quad V = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_0^r$$

$$or \quad V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r} \quad \text{......................... (3)}$$

(b) When point lies on the surface of sphere

When the point P lies on the surface of the sphere $i.e.,\ r = R$,the potential is given by

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R} \quad \text{......................... (4)}$$

(c) When the point lies inside the sphere

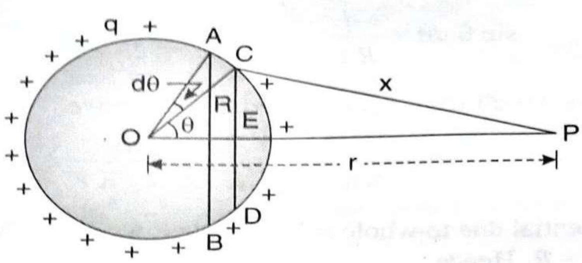
When the point P lies inside the sphere $i.e.,\ (r < R)$, then the potential inside the sphere is the same as on the surface of the sphere. In this case

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R} \quad \text{......................... (5)}$$

POTENTIAL DUE TO CHARGED SPHERICAL CONDUCTOR

(i) When point P lies outside the sphere

Consider a conducting charged spherical conductor with centre O and radius R. Let **σ** be the surface charge density and the total charge be q. When a conducting sphere is given charge, the whole is distributed uniformly on the surface of the sphere and there will be no charge inside the sphere. Now the problem is to find out the potential at the externRa2l point P distant r from the centre of th spherical conductor, For this purpose, we divide the sphere into a



number of rings with centres on OP. Further consider one such ring $ABCD$ between two planes AB and CD. Let $CP = x$, $\angle COP = \theta$ and $\angle AOC = d\theta$.

From the right angled triangle OEC,

$$CE = OC\, sin\theta = R\, sin\theta$$

From sector AOC, $AC = R\, d\theta$

$$The\ circumference\ of\ the\ ring = 2\pi \times (R\, sin\theta)$$

$$\therefore \quad Area\ of\ the\ ring = 2\pi R\, sin\theta \times R d\theta$$

$$= 2\pi R^2 sin\theta\, d\theta$$

$$\therefore \quad Charge\ on\ the\ ring = area\ of\ the\ ring \times surface\ density$$

$$= 2\pi R^2 sin\theta\, d\theta \times \sigma$$

Where $\sigma = \frac{total\ charge\ on\ shell}{total\ surface\ area} = \frac{q}{4\pi R^2}$

$$\therefore \quad charge\ on\ the\ ring \qquad dq = 2\pi R^2\, sin\,\theta\, d\theta \times (\frac{q}{4\pi R^2})$$

$$= \frac{q\, sin\theta\, d\theta}{2} \qquad ....\ (1)$$

So the potential at P due to the charge on the ring

$$dV = \frac{1}{4\pi\varepsilon_0} . \frac{dq}{x} \qquad ....\ (2)$$

$(Every\ point\ of\ the\ narrow\ ring\ \ ABCD\ is\ at\ the\ same\ distance\ x\ from\ the\ point\ P)$

$$dV = \frac{q\, sin\,\theta\, d\theta}{8\pi\, \varepsilon_0\, x} \qquad ....\ (3)$$

From figure, $x^2 = R^2 + r^2 - 2R\, r\, cos\,\theta$

Differentiating this equation, we get

$$2x\, dx = 2\, R\, r\, sin\theta\, d\theta$$

$$or \qquad sin\theta\, d\theta = \frac{x\, dx}{R\, r} \qquad ....\ (4)$$

Substituting the value of $sin\theta\, d\theta$ from eq (4) in eq (3), we have

$$dV = \frac{q\, x\, dx}{8\pi\, \varepsilon_0\, R\, r\, x} = \frac{q\, dx}{8\pi\, \varepsilon_0\, R\, r} \qquad ....\ (5)$$

In order to obtain the potential due to whole spherical shell we integrate the above equation within the limits $x = r - R$ and $x = r + R$.Hence,

$$V = \int_{r-R}^{r+R} dV = \int_{r-R}^{r+R} \frac{q\, dx}{8\pi\, \varepsilon_0\, R\, r} = \frac{q}{8\pi\, \varepsilon_0\, R\, r}\, [x]_{r-R}^{r+R}$$

$$= \frac{q}{8\pi\, \varepsilon_0\, R\, r}\, [r + R - r + R] = \frac{q}{8\pi\, \varepsilon_0\, R\, r} \times 2R$$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0} . \frac{q}{r} \qquad ....\ (6)$$

This expression shows that the potential at an external point is the same as if the total charge on the shell were concentrated at the centre. Further, the potential decreases with distance in inverse proportion.

(i i) When P lies on the surface.

In this case $r = R$

$$Potential\ at\ the\ surface = \frac{1}{4\pi\varepsilon_0} . \frac{q}{r} \qquad ....\ (7)$$

(i i i) When P lies inside the sphere.

In this case, $V = \frac{1}{4\pi\varepsilon_0} . \frac{q}{r} \qquad ....\ (8)$

Thus, the potential at an internal point is the same as that on the surface.

The variation of potential with distance is shown in fig.