Q: State Faraday's laws of electromagnetic induction.

FARADAY'S LAWS

1). Whenever the magnetic flux linked with a circuit, is changed, it gives rise to an induced electromotive force (emf) in the circuit

2) The magnitude of the induced e.m.f is directly proportional to the negative time rate of change of the magnetic flux linked with the circuit

$$\varepsilon = -\frac{d\phi_B}{dt}$$

Here, ε = induced electro motive force (emf)

ϕ_B = magnetic flux

$\frac{d\phi_B}{dt}$ = time rate of change of magnetic flux

If there are N turns in the coil, then

$$\varepsilon = -N\frac{d\phi}{dt}$$

Q Obtain the Integral form of faraday's law

Integral form of faraday's law

Consider a magnetic field, which is oduced by a current carrying coil. Suppose there is a closed circuit C, which encloses a surf in the magnetic field. Let $\bar{B}$ be the magnetic flux density. The magnetic flux through a small area ds will be $\bar{B}\cdot\bar{ds}$

Now, the total magnetic flux through the entire circuit is

$$\phi_B = \oint_S \bar{B}\cdot\bar{ds} \quad \text{---} (1)$$

We know that the emf will be equal to the line integral of the electric field E and hence

$$\mathcal{E} = \oint_l \bar{E}\cdot\bar{dl} \quad \text{---} (2)$$

According faraday's law, induced emf

$$\mathcal{E} = -\frac{d}{dt}(\phi_B)$$

$$= -\frac{d}{dt}\oint_S \bar{B}\cdot\bar{ds}$$

$$\mathcal{E} = -\int \frac{\partial B}{\partial t}\cdot ds \quad \text{---} (3)$$

from equation (2) and (3)

$$\oint_l \bar{E}\cdot\bar{dl} = -\int_S \frac{\partial B}{\partial t}\cdot ds \quad \text{---} (4)$$

Integral form of faraday's law $\boxed{\oint_l \bar{E}\cdot\bar{dl} = -\int_S \frac{\partial B}{\partial t}\cdot ds}$

2. Obtain the differential form of Faraday's law.

Differential form of Faraday's law

Integral form of araday's law is

$$\oint_l \bar{E}\cdot \bar{d} = -\frac{d}{dt}\int_S \bar{B}\cdot \bar{ds}$$

$$= -\int \frac{\partial B}{\partial t}\cdot ds \quad \text{--- (1)}$$

using stoke's theorem L.H.S. of equation (1) is written as

$$\oint_l \bar{E}\cdot \bar{dl} = \int_S (\nabla \times E)\cdot ds \quad \text{--- (2)}$$

From equation (1) and (2)

$$\oint_S (\nabla \times E)\cdot ds = -\int_S \frac{\partial B}{\partial t}\cdot ds$$

$$\oint_S (\nabla \times E)\cdot ds \quad \int_S \frac{\partial B}{\partial t}\cdot ds = 0$$

$$\oint_S \left[(\nabla \times E) + \frac{\partial B}{\partial t}\right] ds = 0$$

$$\nabla \times E + \frac{\partial B}{\partial t} = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Differential form of Faraday's law $\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$

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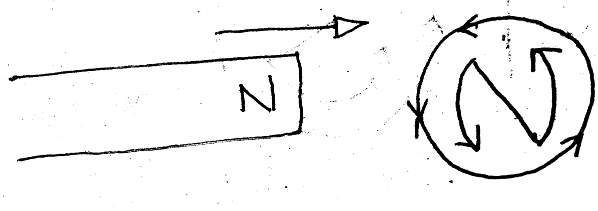
Q: state and explain Lenz's law.

Lenz's law: The direction of the induced emf (or induced current) in a circuit is such that it always opposes the original cause that produces it.

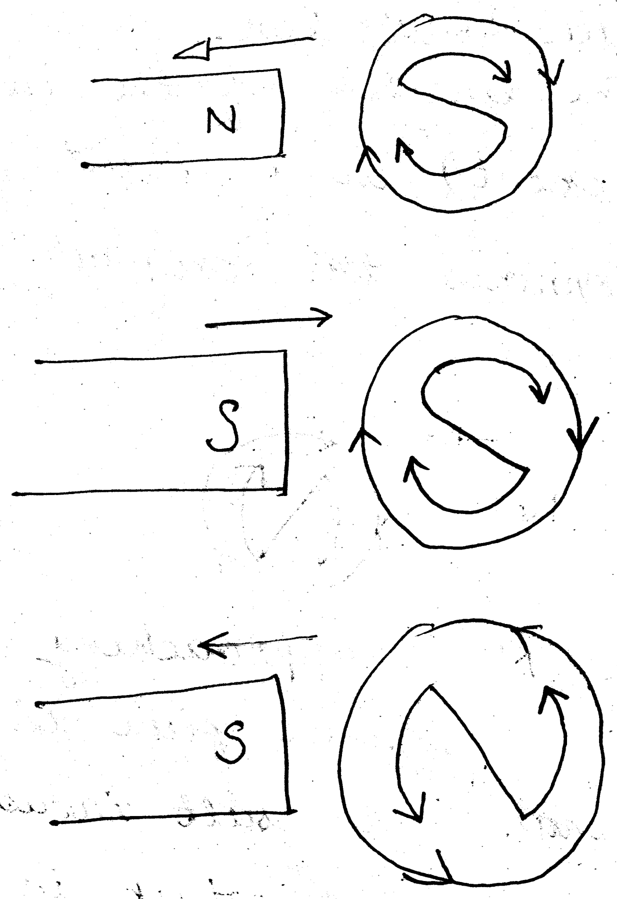


The north pole N is approaching the coil and thereby the magnetic flux through the coil is increasing. As a result induced emf and induced current are set up in the coil.

The cause for induced e.m.f. and induced current is approach of North pole towards the coil.

The induced current should be in such a direction that the motion of the North pole towards the coil is opposed. This is possible when there arises a North pole on the left face of the coil. The left face of the coil will become north pole, when current flows in the anti-clock wise direction in it.

Hence, when the north pole is approaching the coil, the induced current in the coil will be in the anti-clock wise direction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Q. Define self - induction.
Derive an equation for the coefficient of self induction.

A. SELF INDUCTION:

The phenominon of production of an induced emf in a circuit itself due variation of current through the same circuit (increasing or decreasing) is called 'self - induction'.

COEFFICIENT OF SELF INDUCTION

The total magnetic flux 'ϕ' threading a coil is directly proportional to the current 'i' passing through the coil.

That is $\phi \propto i$

or $\phi = L i$ ——— (1)

The constant of proportionality L, is called the "coefficient of self induction" or "self induction"

From equation (1), ~~the~~ coefficient of self induction

$$L = \frac{\phi}{i} \quad \text{———} \quad (2)$$

From equation (2), it follows that when $i = 1$ the coefficient of self induction L is numerically equal to magnetic flux ϕ.

From Faraday's law of induction, the induced e.m.f. (ε) is given by

$$\varepsilon = -\frac{d\phi}{dt} \quad \text{--------} \quad (3)$$

Substituting the value of ϕ from equation (
in equation (3) we get

$$\varepsilon = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$$

$$\text{or } L = \frac{\varepsilon}{-\left(\frac{di}{dt}\right)}$$

The coefficient of self induction is numerically equal to the induced e.m.f., when the current in the coil decreases at unit rate.

The S.I. unit of self induction is

he ry or $\frac{\text{weber}}{\text{ampere}}$ or $\frac{\text{volt - second}}{\text{ampere}}$

Q. Define Mutual induction.

Ans. Derive an expression for the coefficient of Mutual induction.

MUTUAL INDUCTION

Whenever there is a change in current in one coil, there will be an induced e.m.f and current in neighbouring coil which is placed near the first coil. This phenomenon is called 'mutual induction'

The first coil in which current changes is called primary coil and the neighbouring coil in which the e.m.f. is induced is called the 'secondary coil.

It is observed that the flux ϕ_B linked with the secondary coil is proportional to the current in the primary coil.

That is $\phi_B \propto i$

$$\phi_B = Mi \quad \text{——— (1)}$$

Here the constant of proportionality 'M' is called the "coefficient of Mutual induction' or 'Mutual inductance" of the two coil.

The emf ε induced in the secondary coil is given by $\varepsilon = -\frac{d\phi_B}{dt}$

$$\varepsilon = -\frac{d}{dt}(Mi) \quad \left[\because \phi_B = Mi\right]$$

$$= -M\frac{di}{dt}$$

or $M = \frac{\mathcal{E}}{-\left(\frac{di}{dt}\right)}$

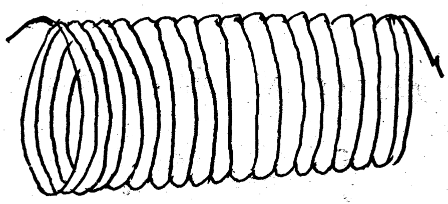
The coefficient of mutual induction ~~is~~ between two c is numerically equal to the magnetic flux linked with the secondary coil, ~~due~~ when ~~to~~ the ~~flow~~ current in the primary coil, decays at unit rate.

The S.I. unit of mutual induction is 'henry'

$$\text{henry} = \frac{\text{weber}}{\text{ampere}} = \frac{\text{volt - second}}{\text{ampere}}$$

Q. Derive an expression for the self-induction of a solenoid.

A solenoid is a long, tightly wound helical coil of wire.



The length of the solenoid l, is assumed to be much greater than the diameter $d (= 2r)$ of the solenoid.

Let n be the number of turns per unit length of the solenoid. The total number of turns will be $\boxed{N = nl}$. When a current 'i' flows through the solenoid, it gives rise to a magnetic field of induction B inside the solenoid.

Then the magnetic induction

$$B = \mu_0 n i \quad \text{——— (1)}$$

The magnetic flux threading through each turn is given by

$$\phi = B \times A$$
$$= (\mu_0 n i)A = \mu_0 n i A$$

The total flux linked with the entire solenoid of $N = nl$ turns is given by

$$\Phi = (\mu_0 n i A) nl = \mu_0 n^2 i A l \quad \text{——— (2)}$$

But $\Phi = Li$ —————— (3)

where L is the self inductance of the solenoid.

$\therefore$ Self inductance of the solenoid

$$L = \frac{\Phi}{i} = \frac{\mu_0 n^2 i A l}{i}$$

or $$\boxed{L = \mu_0 n^2 A l} \quad \text{—————— (4)}$$

or $$\boxed{L = \frac{\mu_0 N^2 A}{l}} \qquad \left[\because n^2 = \frac{N^2}{l^2}\right]$$

$\therefore$ Self inductance of a solenoid

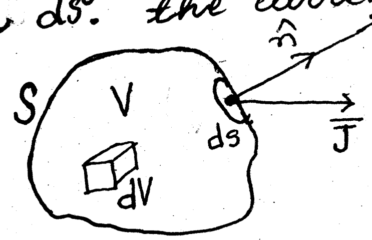
$$\boxed{L = \frac{\mu_0 N^2 A}{l}} \quad \text{—————— (5)}$$

Q. Derive the equation of continuity.

The differential equation relating the current density $\bar{J}$ and volume charge density ρ is known as equation of continuity. This equation is based on the law of conservation of charge i.e. 'the charge can neither be created nor destroyed'.

So, the rate of decrease of total charge inside any arbitrary volume must be equal to the net rate of flow of charge from the volume.

Consider an irregular volume V, enclosed by a surface S. Let $\hat{n}$ be a unit vector perpendicular to a small area ds on the surface S, and $\bar{J}$ be the current density through ds. the current di through ds is given by



$$di = \bar{J} \cdot \hat{n}\, ds = \bar{J} \cdot \overline{ds}$$

∴ The Current i through the whole surface

$$i = \int_S \bar{J} \cdot \overline{ds} \quad \text{———} \ (1)$$

Now consider a small elementary volume dV inside the surface. Let ρ be the volume charge density i.e. charge per unit volume. Then the charge dq in the elementary volume dV is given by

$$dq = \rho\, dV$$

∴ The charge in the whole volume $q = \int_V \rho\, dV$

So, the rate of decrease of charge $\frac{dq}{dt} = -\frac{\partial}{\partial t}\int_V \rho\, dV$ ——— (2)

According to the law of conservation charge equation (1) should be equal to equation (2)

Hence, $\int_S \vec{J}.\vec{ds} = -\frac{\partial}{\partial t}\int_V \rho\, dV$ ——— (3)

According to Gauss Divergence theorem

$$\oint_S \vec{J}.\vec{ds} = \int_V (\nabla.J)\, dV$$

Substituting this in equation (3), we get

$$\int_V (\nabla.J)\, dV = -\frac{\partial}{\partial t}\int_V \rho\, dV$$

$$= -\int_V \frac{\partial \rho}{\partial t}\, dV$$

or $\int_V (\nabla.J)\, dV + \int_V \frac{\partial \rho}{\partial t}\, dV = 0$

or $\int_V \left(\nabla.J + \frac{\partial \rho}{\partial t}\right) dV = 0$ ——— (4)

As volume dV is arbitrary, the integrand must be zero

i.e $\boxed{\nabla.J + \frac{\partial \rho}{\partial t} = 0}$

This is known as equation of continuity

For steady currents $\frac{\partial \rho}{\partial t} = 0$. Then equation of continuity takes the form

$$\nabla.J = 0$$

Q. Derive the modified Ampere's law.

Ampere's law in vector differential form can be expressed as $\boxed{\bar{\nabla}\times\bar{B} = \mu_0\bar{J}}$ ——— (1)

where $\bar{J}$ is current density

We know that the divergence of curl of a vector is zero.

Applying this to the vector $\bar{B}$

we have $\bar{\nabla}\cdot(\bar{\nabla}\times\bar{B}) = 0$

$\Rightarrow \bar{\nabla}\cdot(\mu_0\bar{J}) = 0$ $[\because \nabla\times B = \mu_0 J]$

$\Rightarrow \boxed{\nabla\cdot J = 0}$ ——— (2)

But the equation of continuity states that

$$\nabla\cdot J + \frac{\partial\rho}{\partial t} = 0 \quad \text{——— (3)}$$

where ρ represents the charge density.

As $\nabla\cdot J = 0$, we have

$$\frac{\partial\rho}{\partial t} = 0 \quad \text{——— (4)}$$

$\rho =$ constant

consequently, Ampere's law in the form of $\nabla\times B = \mu_0 J$ or $\nabla\cdot J = 0$ is valid only for a steady condition.

According to Maxwell, Ampere's law in the form of $\nabla\cdot J = 0$ is insufficient for the case of time-varying electrical fields, in which, the charge density ρ varies with time.

Write 25 lines on each page

(i.e $\frac{\partial \rho}{\partial t} \neq 0$ or ρ is not constant).

Maxwell assumed that the definition for current density must include another additional term J_d such that the continuity equation is satisfied even with time varying electric fields.

that is $\bar{\nabla} \cdot (J + J_d) = 0$

$$\nabla \cdot J + \nabla J_d = 0 \qquad (5)$$

But the general continuity equation is

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0 \qquad [3]$$

from equations ~~(3) and~~ (5) we get,

$$\nabla J_d = \frac{\partial \rho}{\partial t} \qquad (6)$$

But Gauss's law in electrostatics, when written in vector differential form $\nabla \cdot D = \rho$ — (7)

from equations (6) and (7) we get $\nabla \cdot J_d = \frac{\partial}{\partial t}(\nabla \cdot D) = \nabla \cdot \frac{\partial D}{\partial t}$

$$J_d = \frac{\partial D}{\partial t}$$

J_d is the displacement current density

The displacement current i_d is given by

$$i_d = \int J_d \cdot ds = \int \frac{\partial D}{\partial t} \cdot ds = \frac{\partial}{\partial t} \int D \cdot ds = \frac{\partial}{\partial t} \int \epsilon_0 E \cdot ds \quad [\because D =$$

$$i_d = \epsilon_0 \frac{\partial}{\partial t} \int E \cdot ds = \epsilon_0 \frac{\partial}{\partial t}(\phi_E)$$

$$\boxed{i_d = \epsilon_0 \frac{\partial \phi_E}{\partial t}}$$

Ampere's law in modified form
or Maxwell - Ampere law is

$$\int B.dl = \mu_0 (i + i_d)$$

or $$\boxed{\int B.dl = \mu_0 i + \epsilon_0 \frac{\partial \phi_E}{\partial t}}$$

Modified Ampere's law (Maxwell - Ampere law) in differential form is

$$\nabla \times B = \mu_0 (J + J_d)$$

or $$\nabla \times B = \mu_0 \left(J + \frac{\partial D}{\partial t}\right)$$